Initial Linac for a Neutrino Factory

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The initial linac after cooling for a neutrino factor has a problem in that the longitudinal emittance is larger than a bucket with the minimum possible frequency can accept. Various schemes are discussed for minimizing the dynamic losses in the linac.

I. REVIEW

A Linearization

From [1], the square of the aspect ratio for a matched distribution is

$$\frac{\sigma_\Delta^2}{\sigma_\tau^2} = -\frac{qcv\omega(p_0c)^3\sin\psi}{(mc^2)^2} \tag{1}$$

and the square of the synchrotron wave number is

$$k_s^2 = -\frac{qv\omega(mc^2)^2\sin\psi}{c(p_0c)^3}. \tag{2}$$

Here q is the charge in a particle, c is the speed of light, v is the average accelerating gradient on crest, ω is the RF frequency, p_0 is the reference momentum, m is the particle's mass, and ψ is the phase of the RF measured from the crest. The coordinates are τ, the deviation of the arrival time from the arrival time of a reference particle, and Δ, the deviation of the energy from the energy of a reference particle.

When $\cos\psi = 0$, the particles are not accelerated, and the linear Hamiltonian in [1] can be integrated when $\psi = -\pi/2$ to give the transfer map

$$\begin{bmatrix} \cos k_sL & \beta_s\sin k_sL \\ -\beta_s^{-1}\sin k_sL & \cos k_sL \end{bmatrix} \tag{3}$$

where the length of the linac is L, the coordinates are (τ, $-\Delta$), and

$$\beta_s = \sqrt{\frac{(mc^2)^2}{qvc\omega(p_0c)^3}}. \tag{4}$$

B Nonlinear Bucket

For an RF bucket which is matched to a bunch of emittance ϵ_L such that the full width of the bucket in the energy direction is $2k\sigma_\Delta$, the following relationship holds:

$$4\frac{p_0c}{\omega}\sqrt{\frac{qvp_0c^2}{(mc^2)^2\omega}}\frac{\psi\cos\psi-\sin\psi}{\sqrt{-\sin\psi}}\left[1+\frac{qvp_0c^2}{(mc^2)^2\omega}(\psi\cos\psi-\sin\psi)\right] = k^2\epsilon_L. \tag{5}$$

C Decays

For a bunch at constant energy, the number of particles as a function of longitudinal position s is given by

$$N(s) = N(0)e^{-ms/p\tau} \tag{6}$$

where p is the momentum of the particles and τ is the decay lifetime of the particles. If the particles are undergoing acceleration at constant gradient, the result can be written in terms of initial and final energy/momentum as [1]

$$N_f = N_i\left(\frac{E_f+p_fc}{E_i+p_ic}\right)^{-\frac{mc}{\tau qv\cos\psi}}. \tag{7}$$

Here the f subscripts refer to the final values, and the i subscripts refer to the initial values.

II. PHASE SPACE ROTATION

If the matrix Σ is defined to be

$$\Sigma(s) = \int \boldsymbol{x}\boldsymbol{x}^T\rho(\boldsymbol{x},s)\,d\boldsymbol{x} \tag{8}$$

where $\rho(\boldsymbol{x},s)$ is the phase space distribution at the independent coordinate s in the phase space coordinates $\boldsymbol{x}$, then this matrix transforms under the linear transfer map $M(s',s)$ as

$$\Sigma(s') = M(s',s)\Sigma(s)M^T(s',s). \tag{9}$$

For an upright longitudinal ellipse,

$$\Sigma = \begin{bmatrix} \sigma_\tau^2 & 0 \\ 0 & \sigma_\Delta^2 \end{bmatrix}. \tag{10}$$

To transform one upright ellipse with RMS parameters σ_τ and σ_Δ to another upright ellipse with RMS paramters σ'_τ and σ'_Δ, using a transfer map of the form (3) requires that $\cos k_s L = 0$ and

$$\beta_s = \frac{\sigma_\tau}{\sigma'_\Delta} = \frac{\sigma'_\tau}{\sigma_\Delta} \tag{11}$$

As an example which we will use later, assume a beam with momentum p_0 and RMS sizes σ_τ and σ_Δ is to be matched into an accelerating linac with frequency ω, gradient v, and phase ψ. Then the gradient v' of the phase rotation linac with frequency ω' that precedes the accelerating linac should be

$$qv' = \frac{\sigma_\Delta}{\omega'\sigma_\tau}k_s, \tag{12}$$

and the length of that linac should be

$$L = \frac{\pi}{2}\sqrt{\frac{c(p_0c)^3}{qv'\omega'(mc^2)^2}}. \tag{13}$$

Here k_s is the value for the synchrotron wave number from Eq. (2), with the values for the accelerating linac following the phase rotation being used in that formula.

Equations (12–13) give the rather surprising result that the length of the phase rotation linac is independent of the choice of the frequency of that linac. There is a tradeoff here: a higher frequency linac is advantageous in that it presumably has a lower hardware cost per unit length, and since the gradient is lower and the frequency is higher, the power requirements associated with that linac should be smaller. On the other hand, Eq. (5) indicates that for a lower RF frequency, the area of the bucket will be larger, and therefore one can expect that the nonlinear distortion of the bunch will be less.

In principle, the phase space rotation could be carried out while accelerating, potentially reducing the required length of the linac from having acceleration and phase rotation sections separated. This possibility will not be treated in this paper.

III. ACCELERATION SCENARIO

We now compute an acceleration scenario for the initial linac. The scenario will consist of three steps:

1. Phase rotate the bunch without accelerating so that it is matched to the next acceleration stage.

2. Accelerate the bunch at constant phase ψ_0 until k, the ratio of the bucket half-width to the RMS energy spread, becomes 4 (some other choice could be made as well).

3. Continue to accelerate, adjusting the RF phase so that k remains 4.

TABLE I: Neutrino factory parameters relevant to initial linac.

Initial momentum	p_i	190 MeV/c
Initial energy spread	$\sigma_{\Delta,i}$	18.3 MeV
Initial bunch length	$\sigma_{\tau,i}$	457 ps
RF frequency	ω	200 MHz
Maximum gradient	$v_{\max}$	15 MV/m
Linac filling factor		0.65

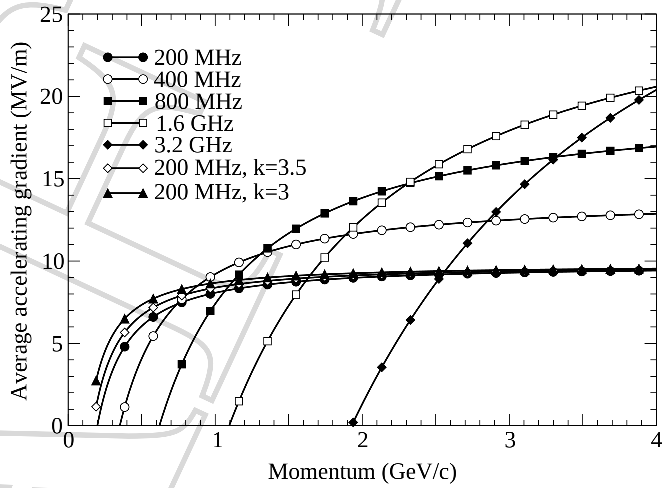

FIG. 1: Accelerating gradient versus momentum for various RF frequencies. Curves are for $k = 4$ unless otherwise specified.

We will plot the following quantities as a function of ψ_0:

- The decay losses
- The factor k during the phase rotation
- The factor k at the beginning of the constant-phase acceleration
- The lengths of each stage, and the total length of the accelerator.

The parameters used are summarized in Tab. I.

A Frequency Transitions

We would like to accelerate to 4 GeV before going into the recirculators due to difficulties in arc design at lower energies resulting from the large energy spread in the beam [2]. One option is to use a single 200 MHz linac all the way to 4 GeV. However, this is probably not optimal due to the lower gradient that can be expected at lower frequencies, as well as the higher costs associated with lower frequency linacs. Hence we would like to have a transition from a 200 MHz linac to a higher frequency linac at some point. Using the technique described in [1], we can produce the curves of accelerating gradient

TABLE II: Energies at which transitions between different linac frequencies should occur. f_{in} is the frequency before the transition, k_{in} is the k value before the transition, f_{out} is the frequency after the transition, and p is the momentum at which the transition occurs.

f_{in} MHz	k_{in}	f_{out} MHz	p MeV/c
200	4	400	738
200	3.5	400	812
200	3	400	880
200	4	800	1067
200	3.5	800	1097
200	3	800	1125
400	4	800	1305
400	4	1600	1885
800	4	1600	2306
800	4	3200	3323

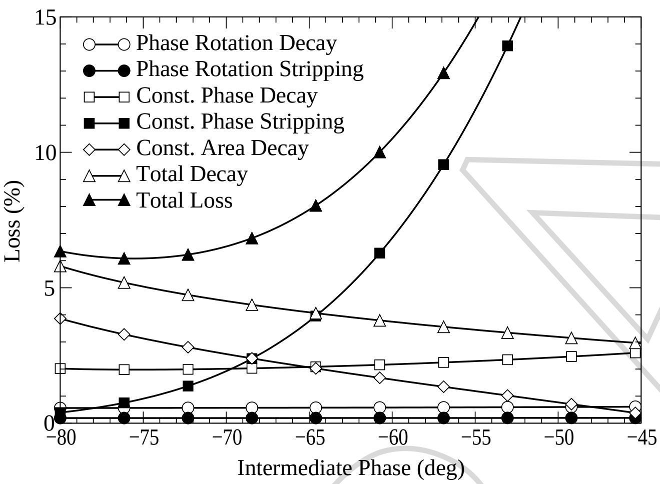

FIG. 2: Losses in the 200 MHz linac.

versus momentum shown in Fig. 1. The momentuma at which transitions occur can be read off from the graph, and are summarized in Tab. II. Note that the figure and table assume that the maximum gradient scales as

$$v_{\max} = 30\ \text{MV/m}\sqrt{\frac{f}{800\ \text{MHz}}}. \tag{14}$$

B The 200 MHz Linac

Using Tab. II, one might consider having a transition from a 200 MHz linac to a 400 MHz linac at 738 MeV/c, assuming that you used $k = 4$ for both linacs. Thus, for this part of the study we will consider 738 MeV/c as the maximum energy to which we accelerate.

Using the techniques described above, a scenario can be worked out for accelerating to 738 MeV/c in a 200 MHz linac. Figure 2 shows the losses in the 200 MHz linac. As expected, total losses from decays decrease

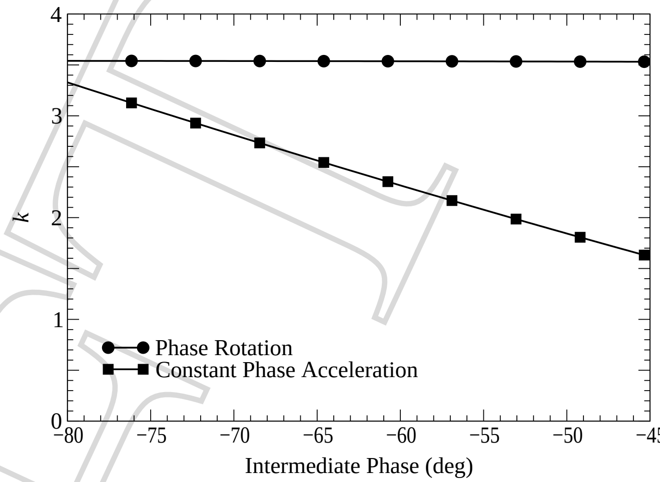

FIG. 3: k values for the phase rotation and constant-phase accelerating sections in the 200 MHz linac.

as the phase of the constant-phase acceleration stage moves toward crest. Similarly, the losses due to the RF bucket stripping out high-amplitude particles increase as the phase moves toward crest. The losses from stripping were computed under the simplistic assumption that all the particles beyond a certain amplitude are lost. That amplitude is assumed to be at $k\sigma$ in each direction in longitudinal phase space; this corresponds to an assumption that the loss is $e^{-k^2/2}$. The stripping loss in each section is computed based on the k values from Fig. 3. Computing the total loss based on this, it appears that the minimum loss occurs for a phase of about 75° in the constant phase acceleration.

Some interesting features of Figs. 2 and 3 are that

- The behavior in the phase rotation section is almost completely independent of the intermediate phase chosen.
- The decay losses in the constant phase section are independent of the phase chosen. The primary effect of the phase is to determine how much of the beam is lost dynamically due to being outside the bucket.
- The chosen phase has a strong effect on the decay losses in the constant-area accelerating section.

In reality, the assumptions about the number of particles lost to dynamic losses may be pessimistic. The reason is that the RF bucket rapidly opens up as the bunch is accelerated, and thus the bunch will go through only a fraction of a synchrotron oscillation with a small area. On the other hand, the changing shape of the bucket also mean that there will be emittance growth and/or losses due to the fact that the bucket is not so well-defined as the adiabatic approximation would indicate. In addition, the fact that the adiabatic approximation is not completely valid will mean that in practice the bunch

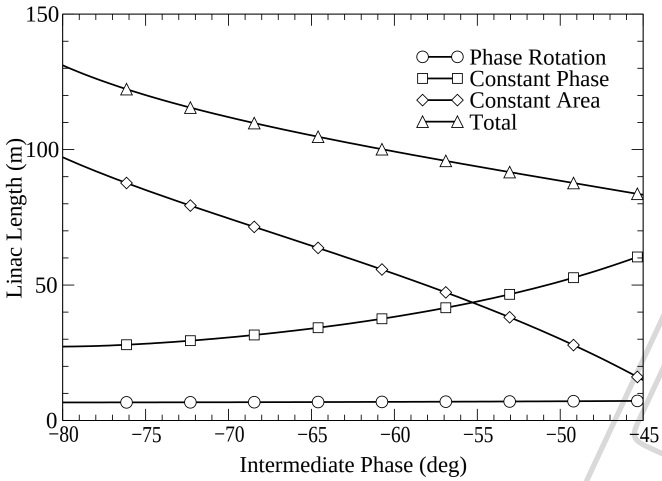

FIG. 4: Length of the 200 MHz linac for various portions of the linac.

TABLE III: Acceleration scenarios

Type	Frequency MHz	Gradient MV/m	Length m	Loss %	k
Constant Area	200	9.75	449.8	8.5	4.0
Constant Area	200	9.75	85.1	5.6	4.0
Phase Rotation	400	6.10	41.7	6.4	6.1
Constant Area	400	13.79	285.1	8.8	4.0
Constant Area	200	9.75	85.1	5.6	4.0
Phase Rotation	400	6.10	41.7	6.4	6.1
Constant Area	400	13.79	62.0	7.4	4.0
Phase Rotation	800	8.64	58.2	8.0	6.1
Constant Area	800	19.50	183.7	9.2	4.0
Constant Area	200	9.75	126.8	6.3	4.0
Phase Rotation	800	4.84	57.5	7.1	4.4
Constant Area	800	19.50	209.1	8.6	4.0

may effectively be mismatched, giving emittance growth and/or loss. Thus, the assumption that the bunch is stripped by the edges of the adiabatic bucket at the lowest energy in the constant-phase section may be a good compromise working assumption.

The dependence of the length of various portions of the linac on the intermediate phase is shown in Fig. 4. Since the cost of the linac is primarily determined by its length, there may be an advantage to choosing a solution with greater dynamic losses in exchange for cost savings, particularly near the minimum in loss where a small change in phase results in a small change in loss, but will result in a larger relative change in the cost.

C Multiple Linac Frequencies

Now consider various scenarios for accelerating up to 4 GeV. All have the same common phase rotation (200 MHz, 6.7 m, 8.1 MV/m) and constant-phase acceleration at 75° (200 MHz, 28.3 m, 9.75 MV/m) described in the previous section. The decay loss at the end of these sections is 2.5%. Table III shows the various scenarios, giving the length of each section, RF parameters in each section, the cumulative decay loss at the end of each section, and the value of k in each section.

Examining the scenarios, note that the minimum decay loss occurs when only one RF frequecy is used. The reason for this is that the decay losses in the phase rotations are not made up for by the higher accelerating gradients in the higher frequency linacs. As the higher frequency (and therefore gradient) linacs get longer, the difference in gradients will eventually make up the difference, but with a maximum momentum of 4 GeV/c, the optimum configuration for decays is still to use a single 200 MHz linac.

However, for cost, the use of a single 200 MHz linac is probably not optimum. For instance, the combination of a 200 MHz and a 800 MHz linac is over 50 m shorter than the 200 MHz linac by itself. In addition, the higher-frequency linac may have lower costs per unit length associated with it: lower RF power requirements, less heating, and lower structure costs (due to size differences). The optimum choice from a cost point of view depends on how the cost of the RF system depends on frequency. The dependency of the decay loss on the scenario is sufficiently weak that the cost should probably be considered in preference to decay loss in optimization of the scenario.

However, dynamic considerations should come into play in addition to cost. One of the primary factors to consider here is the parameter k in the phase rotation. Since the phase rotation involves a beam mismatched to the bucket, a smaller k will lead to an emittance blowup in the phase rotation. One expects that the value of 4.4 required for the phase rotation between the 200 MHz and 800 MHz linac in that two-frequency scenario is too small; this is a relatively full bucket. The value of 6.1 used in the other scenarios is probably more reasonable. Note that in all cases, using the lower-frequency linac to do the phase rotation instead of the higher-frequency linac is impossible: the required gradient would be larger than what we are expecting to be able to achieve (see Eq. (12). Since the bucket area increases with increasing p_0c, one could imagine attempting the transition at a higher momentum to increase the k in the phase rotation. However, this is probably not advantageous: while (using small angle approximations) k is approximately proportional to $(p_0c)^{3/5}$, the length of the phase rotation is approximately proportional to $(p_0c)^{9/5}$, and thus one quickly loses the advantage of putting in the frequency transition in the first place.

A final consideration is the phase space shape of the bunch at the end of the linac. This should preferably be as close to the shape of the phase space desired in the next stage (that of a recirculator) as possible. Table IV shows the bunch lengths and energy spreads in the beam as a function of the energy at the end of the linac. To evaluate which scenario might be best, it is necessary to consider

TABLE IV: Beam sizes for various frequencies.

Frequency MHz	σ_τ ps	σ_Δ MeV
200	60	139
400	43	197
800	30	276

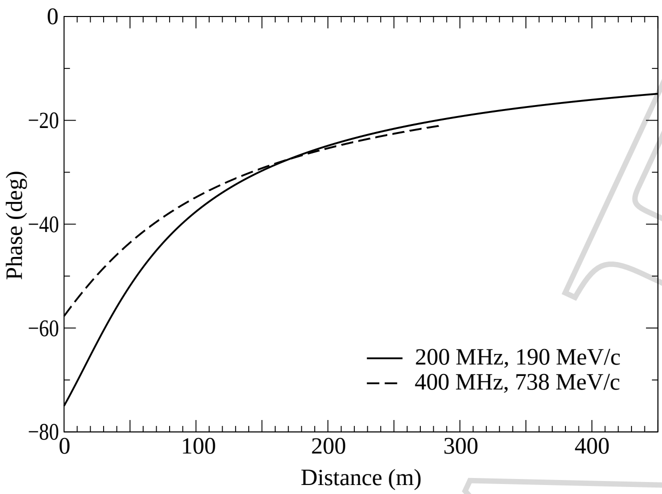

FIG. 5: RF phase as a function of position along the linac. The legend gives the RF frequency and the momentum at the beginning of the linac.

the design of the recirculator. However, note that the beam output by the 200 MHz linac may be considered more attractive since it has the smallest energy spread (a 3.5% relative RMS energy spread!).

Figure 5 gives the RF phase profile along the linac for the 200 MHz and 400 MHz constant-area acceleration scenarios. The above considerations seem to rule out any scenario with 800 MHz RF: with only 200 MHz and 800 MHz, the phase rotation does not seem realistic due to emittance growth; with three frequencies, the total linac length is longer, there are more decays, and one has the added complexity of three RF systems instead of two; in either scenario the energy spread at the end of the 800 MHz linac is exceedingly large.

D Superconducting Example

Here's an example more appropriate for superconducting linacs. Instead of a filling factor of 0.65, use a filling factor of 0.5. The maximum energy we will be concerned with will only be 3 GeV. Figure 6 shows the gradients as a function of momentum, from which we get the transition momentum shown in Tab. V.

The losses in the 200 MHz linac are shown in Fig. 7, leading to an estimate that the optimum phase to run the intermediate constant-phase linac at is 75°. The initial phase rotation ends up being 7.1 m long with a average

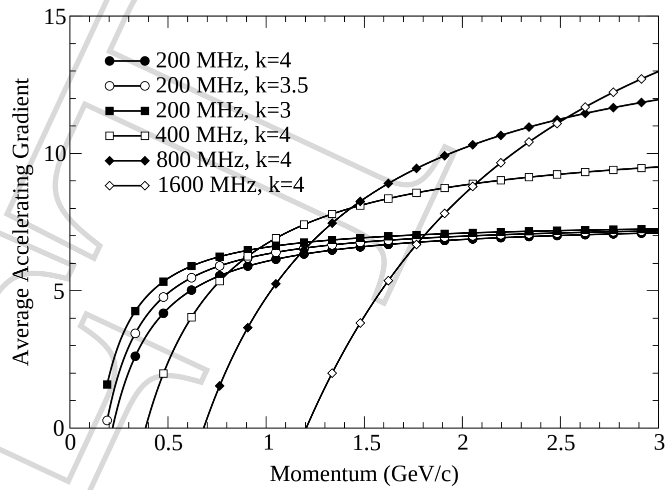

FIG. 6: Average accelerating gradient versus momentum for various RF frequencies with 0.5 filling factor.

TABLE V: Energies at which transitions between different linac frequencies should occur, for 0.5 filling factor.

f_{in} MHz	k_{in}	f_{out} MHz	p MeV/c
200	4	400	809
200	3.5	400	890
200	3	400	964
200	4	800	1171
200	3.5	800	1204
200	3	800	1234
200	4	1600	1777
200	3.5	1600	1794
200	3	1600	1810
400	4	800	1433
400	4	1600	2071
800	4	1600	2534

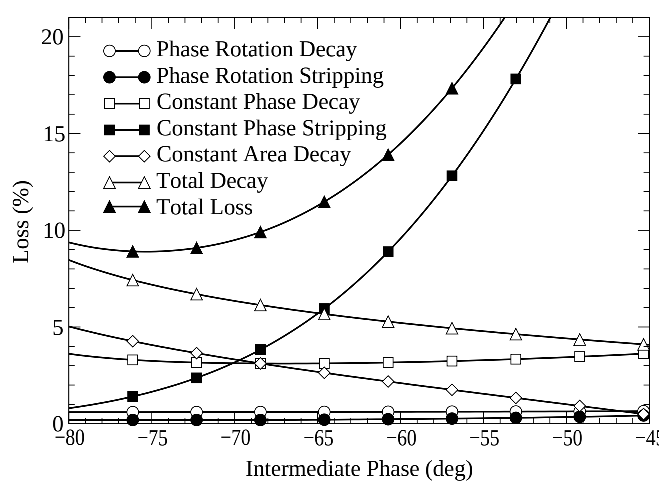

FIG. 7: Losses in the 200 MHz linac with 0.5 filling factor.

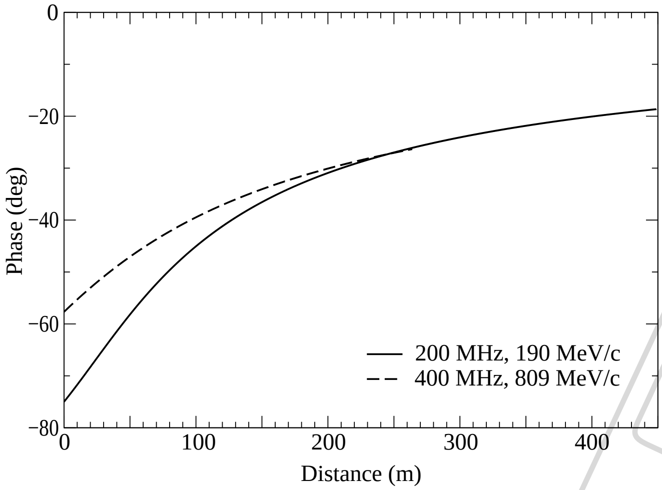

FIG. 8: RF phase as a function of position along the 0.5 filling factor linac. The legend gives the RF frequency and the momentum at the beginning of the linac.

accelerating gradient of 7.08 MV/m (includes filling factor). The constant phase linac is 49.2 m long with a gradient of 7.5 MV/m. From that point, there are two reasonable scenarios: the first is accelerating with 200 MHz at 7.5 MV/m to 3 GeV/c with a linac of length of 448.9 m, for a total decay loss of 10.7% from beginning to end; the second is accelerating with a 200 MHz 7.5 MV/m linac of length 121.8 m to 809 MeV/c, then a phase rotation with 54.6 m of 400 MHz linac at 4.69 MV/m, then a 263.9 m long 400 MHz linac at 10.6 MV/m going up to 3 GeV/c, which gives a total decay loss of 11.1%. Figure 8 gives the phase profile along the linac for the constant-area acceleration part of these scenarios.

The 400 MHz system will almost certainly cost less: the linac is shorter, and the higher frequency linac should be less expensive. In addition, the quadrupoles used for focusing can be smaller. The 200 MHz system has only slightly fewer decays, but may be advantageous from the point of view that a second system is not required.

E Normal Conducting Start

Let's say that instead we can only use normal conducting linac with a real-estate gradient of 12 MV/m up until 320 MeV/c, and after that we switch to superconducting linacs described in the previous subsection. The reason for doing this would be that a FODO lattice will not be able to keep the beam within the beam pipe of a TESLA-scaled 200 MHz superconducting linac until the momentum reaches about 320 MeV/c. In principle the bunch can be accelerated straight off in a $k = 4$ bucket, but the phase at 190 MeV/c is $-88°$, and thus the net acceleration would be extremely small.

Figure 9 shows the loss as a function of the phase of the constant-phase acceleration stage. Again, a phase

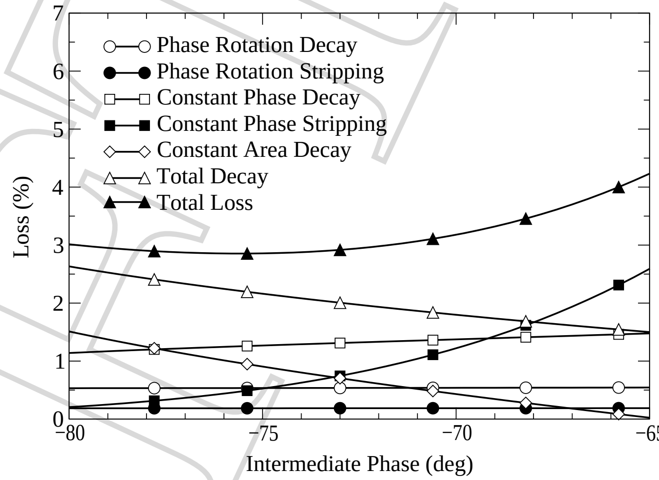

FIG. 9: Losses in the 12 MV/m 200 MHz linac.

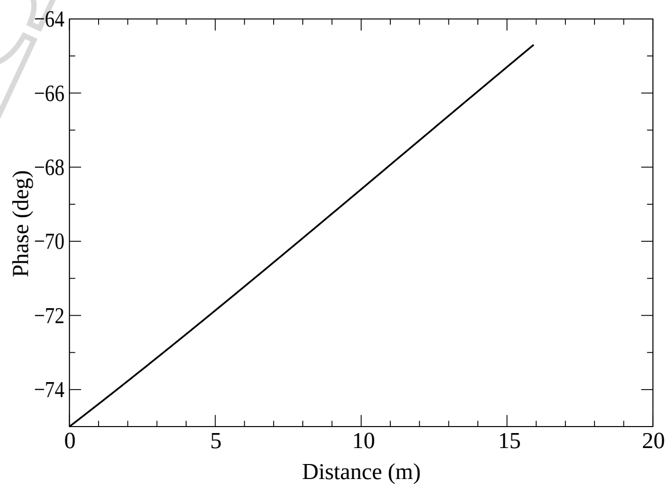

FIG. 10: RF phase as a function of position along the 12 MV/m 200 MHz linac.

of $-75°$ seems to be optimal. The result is a 6.4 m long phase rotation at 8.96 MV/m, followed by acceleration to 250 MeV at 12 MV/m for 17.4 m, followed by acceleration at constant area to 320 MeV at 12 MV/m for 15.9 m, followed by a phase rotation at 8.78 MV/m for 14.0 m, giving a total decay loss of 3.4%. The phase profile for this linac is shown in Fig. 10.

From 320 MeV/c, we switch to superconducting linacs. We again have the two possibilities of continuing on all the way with 200 MHz linac or switching to 400 MHz linac. With exclusively 200 MHz linac, the remaining linac at constant area is 437.8 m long, and the total decay loss for the entire system is 9.7%. For the scenario with 400 MHz linac, there is 110.7 m of 200 MHz linac accelerating to 809 MeV/c, followed by a 400 MHz 4.69 MV/m phase rotation for 54.6 m, followed by 263.9 m of constant-area 400 MHz linac at 10.61 MV/m, accelerating to 3 GeV/c, giving a total decay of 10.2%. The phase profiles in the constant-area linacs can be deduced by taking the appropriate portion of Fig. 8.

This scenario is shorter and has fewer decays (and fewer dynamic losses) than a pure-superconducting system, but will require more power.

REFERENCES

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[1] J. Scott Berg. "Acceleration for a High Energy Muon Collider." To appear in the proceedings of *Studies on Colliders and Collider Physics at the Highest Energies: Muon Colliders at 10 TeV and 100 TeV—HEMC '99*, Montauk, NY, 27 September–1 October 1999. Also BNL report BNL-67115.

[2] Carol Johnstone. Private Communication.